

Test of factorisation in $B \rightarrow K\pi$ decays

Claudia Isola and T. N. Pham

Centre de Physique Théorique,

Centre National de la Recherche Scientifique, UMR 7644,

Ecole Polytechnique, 91128 Palaiseau Cedex, France

(24 November 1999)

Abstract

We analyse the $B \rightarrow K\pi$ decays using the factorisation model with final state interaction phase shift included. We find that factorisation seems to describe qualitatively the latest CLEO data. For a test of the factorisation model, we derive a relation for the branching ratios independent of the strength of the strong penguin interactions. This relation gives a central value of (0.60×10^{-5}) for $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$, somewhat smaller than the latest CLEO measurement, but the experimental errors are yet too big to take it as a real prediction of the factorisation model. We also find that a ratio obtained from the CP-averaged $B \rightarrow K\pi$ decay rates could be used to test the factorisation model and to determine the weak angle γ with more precise data, though the latest CLEO data seem to favor γ in the range of $(90^\circ - 120^\circ)$.

One of the possibilities offered by the $B \rightarrow K\pi$ decays is the determination of the CP-violating phase γ , one of the angles in the (db) unitary triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix in the standard model [1]. Infact the large CP-averaged branching ratio(\mathcal{B}) for $B \rightarrow K\pi$ as observed by the CLEO Collaboration [3] indicate that the penguin interactions contribute a major part to the decay rates and provide an interference between the Cabibbo-suppressed tree and penguin contribution resulting in a CP-asymmetry between the $B \rightarrow K\pi$ and its charge conjugate mode. The CP-averaged decay rates depend also on the weak phase γ and give us a determination of this phase once a reliable description of the $B \rightarrow K\pi$ decays could be established [4,5].

With the latest measurement by the CLEO collaboration [3] , we have now the CP-averaged branching ratios for all the $B \rightarrow K\pi$ decay modes. In particular, the $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ mode is found to have a large branching ratio of $(1.46_{-5.1}^{+5.9+2.4}) \times 10^{-5}$ compared with a value in the range $(0.5-0.74) \times 10^{-5}$ in the factorisation model [6,7]. The predicted values for other modes are, however, more or less in agreement with experiment. As the effective Hamiltonian for $B \rightarrow K\pi$ decays is well established with the short-distance Wilson coefficients for tree and penguin operators now given at the next-to-leading logarithms(NLL) QCD radiative corrections [7–12], the most important theoretical uncertainties would probably come from long-distance matrix elements obtained with the factorisation model and final state interaction (FSI) effects. Infact one of the main uncertainties in the penguin contributions to $B \rightarrow K\pi$ decays come from the value of the current s quark mass which is not known to a good accuracy. There are also non-factorisation terms which must be included in the form of an effective Wilson coefficients to make the amplitudes scale-independent [7,13]. Thus a more precise test of factorisation is to consider quantities which are independent of the strong penguin contributions. This is the main purpose of this paper. When all the $B \rightarrow K\pi$ decay modes are measured with good accuracy, and if the rescattering phase is known the dominant strong penguin contribution could be determined from the measured branching ratios assuming factorisation for the small tree-level and electroweak penguin terms, as will be discussed in the following. Though the present data are not yet sufficiently accurate for a

determination of the effective Wilson coefficients in $B \rightarrow K\pi$ decays at this time, a first step toward an understanding of $B \rightarrow K\pi$ decays is to see how well these penguin-dominated charmless B decays can be described by factorisation using the Wilson coefficients obtained from perturbative QCD. As argued in [14], for these very energetic decays, because of color transparency, factorisation should be a good approximation for $B \rightarrow K\pi$ decays if the Wilson coefficients are evaluated at a scale $\mu = O(m_b)$. We could thus proceed to the test of factorisation bearing in mind that there are possible scale-dependent corrections from non-factorisation terms to be determined with more precise data. To include FSI effects, as in [6], we assume that elastic FSI effects can be absorbed into the two $\Delta I = 1/2$ and $\Delta I = 3/2$ elastic $\pi K \rightarrow \pi K$ rescattering phases δ_1 and δ_3 taken as free parameters and include only inelastic effects coming from the charm and charmless intermediate state contributions to the absorptive part of the decay amplitudes. These inelastic contributions can be included in the Wilson coefficients of the penguin operators which now have an absorptive part and are given in [10,12,15].

We begin by first giving predictions in factorisation model for the $B \rightarrow K\pi$ decay rates and branching ratios in terms of the rescattering phase difference δ and for a typical value of the weak phase γ . As will be seen, factorisation seems to produce sufficient $B \rightarrow K\pi$ decay rates. We could thus proceed to a test of the factorization model by comparing with experiments, quantities obtained by factorisation which are independent of the strong rescattering phase difference [16]. We find that the sum of the CP-averaged branching ratios $\mathcal{B}(B^- \rightarrow K^- \pi^0) + \mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-)$ and $\mathcal{B}(\bar{B}^0 \rightarrow K^- \pi^+) + \mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ are independent of the FSI rescattering phase. Other quantities obtained from various combination of the decay rates, for example, the quantity Δ defined as $\Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(\bar{B}^0 \rightarrow K^- \pi^+) - 2(\Gamma(B^- \rightarrow K^- \pi^0) + \Gamma(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0))$ is independent of the strong penguin contributions and could be used to predict $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ in terms of the other measured branching ratios. As the main purpose of this paper is to test the factorisation model using relations independent of the strong penguin interactions, we will not discuss here a recent theoretical work on factorisation in $B \rightarrow \pi\pi$ decays which should be completed to have all the logarithms of m_b

under control [17].

In the standard model, the effective Hamiltonian for $B \rightarrow K\pi$ decays are given by [8,9,12],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{cs}^*(c_1O_1^c + c_2O_2^c) - \sum_{i=3}^{10} (V_{tb}V_{ts}^*c_i)O_i] + \text{h.c.} , \quad (1)$$

in standard notation. At next-to-leading logarithms, c_i take the form of an effective Wilson coefficients c_i^{eff} which contain also the penguin contribution from the c quark loop and are given in [10,12].

The tree level operators O_1 and O_2 as well as the electroweak penguin operators $O_7 - O_{10}$ have both $I = 0$ and $I = 1$ parts while the QCD strong penguin operators $O_3 - O_6$ have only $I = 0$ terms. The $B \rightarrow K\pi$ decay amplitudes can now be expressed in terms of the decay amplitudes into $I = 1/2$ and $I = 3/2$ final states as [6],

$$\begin{aligned} A_{K^-\pi^0} &= \frac{2}{3}B_3e^{i\delta_3} + \sqrt{\frac{1}{3}}(A_1 + B_1)e^{i\delta_1}, \\ A_{\bar{K}^0\pi^-} &= \frac{\sqrt{2}}{3}B_3e^{i\delta_3} - \sqrt{\frac{2}{3}}(A_1 + B_1)e^{i\delta_1}, \\ A_{K^-\pi^+} &= \frac{\sqrt{2}}{3}B_3e^{i\delta_3} + \sqrt{\frac{2}{3}}(A_1 - B_1)e^{i\delta_1}, \\ A_{\bar{K}^0\pi^0} &= \frac{2}{3}B_3e^{i\delta_3} - \sqrt{\frac{1}{3}}(A_1 - B_1)e^{i\delta_1}, \end{aligned} \quad (2)$$

where A_1 is the sum of the strong penguin A_1^S and the $I = 0$ tree level A_1^T as well as the $I = 0$ electroweak penguin A_1^W contributions to the $B \rightarrow K\pi$ $I = 1/2$ amplitude; similarly B_1 is the sum of the $I = 1$ tree level B_1^T and electroweak penguin B_1^W contribution to the $I = 1/2$ amplitude, and B_3 is the sum of the $I = 1$ tree level B_3^T and electroweak penguin B_3^W contribution to the $I = 3/2$ amplitude.

The factorisation approximation is obtained by neglecting in the Hamiltonian terms which are the product of two color-octet operators after Fierz reordering of the quark fields. The effective Hamiltonian for non-leptonic decays are then given by Eq.(1) with c_j replaced

by a_j and O_j expressed in terms of hadronic field operators. In the notation of Ref. [6], we have

$$\begin{aligned}
A_1^T &= i \frac{\sqrt{3}}{4} V_{ub} V_{us}^* r a_2, \\
B_1^T &= i \frac{1}{2\sqrt{3}} V_{ub} V_{us}^* r \left[-\frac{1}{2} a_2 + a_1 X \right], \\
B_3^T &= i \frac{1}{2} V_{ub} V_{us}^* r [a_2 + a_1 X], \\
A_1^S &= -i \frac{\sqrt{3}}{2} V_{tb} V_{ts}^* r [a_4 + a_6 Y], \quad B_1^S = B_3^S = 0 \\
A_1^W &= -i \frac{\sqrt{3}}{8} V_{tb} V_{ts}^* r [a_8 Y + a_{10}], \\
B_1^W &= i \frac{\sqrt{3}}{4} V_{tb} V_{ts}^* r \left[\frac{1}{2} a_8 Y + \frac{1}{2} a_{10} + (a_7 - a_9) X \right], \\
B_3^W &= -i \frac{3}{4} V_{tb} V_{ts}^* r [(a_8 Y + a_{10}) - (a_7 - a_9) X], \tag{3}
\end{aligned}$$

where $r = G_F f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)$, $X = (f_\pi/f_K)(F_0^{BK}(m_\pi^2)/F_0^{B\pi}(m_K^2))(m_B^2 - m_K^2)/(m_B^2 - m_\pi^2)$, $Y = 2m_K^2/[(m_s + m_q)(m_b - m_q)]$ with $q = u, d$ for $\pi^{\pm,0}$ final states, respectively, and a_j are given in terms of the effective Wilson coefficients c_j^{eff} (N_c is the number of effective colors) by

$$\begin{aligned}
a_j &= c_j^{\text{eff}} + c_{j+1}^{\text{eff}}/N_c \text{ for } j = 1, 3, 5, 7, 9 \\
a_j &= c_j^{\text{eff}} + c_{j-1}^{\text{eff}}/N_c \text{ for } j = 2, 4, 6, 8, 10. \tag{4}
\end{aligned}$$

In our analysis, we use $N_c = 3$ and $m_b = 5.0$ GeV which give a_j the following numerical values

$$\begin{aligned}
a_1 &= 0.07, \quad a_2 = 1.05, \\
a_4 &= -0.043 - 0.016i, \quad a_6 = -0.054 - 0.016i, \\
a_7 &= 0.00004 - 0.00009i, \quad a_8 = 0.00033 - 0.00003i, \\
a_9 &= 0.00907 - 0.00009i, \quad a_{10} = -0.0013 - 0.00003i. \tag{5}
\end{aligned}$$

Note that a_1 is sensitive to N_c and is rather small for $N_c = 3$. As there is no evidence for a large positive a_1 in $B \rightarrow K\pi$ decays which are penguin-dominated and are not sensitive to

a_1 , we use a_1 evaluated with $N_c = 3$ given in Eq.(5). Indeed, the predicted branching ratios remain essentially unchanged with $a_1 = 0.20$ taken from the Cabibbo-favored B decays [18,19].

In the absence of FSI rescattering phases, we recover the usual expressions for the decay amplitudes in the factorisation approximation. We have used c_j^{eff} given at next-to-leading order in QCD radiative corrections [8,9,12] and evaluated at a scale $\mu = m_b$. We note that the coefficients c_3^{eff} , c_4^{eff} , c_5^{eff} and c_6^{eff} are enhanced by the internal charm quark loop due to the large time-like virtual gluon momentum $q^2 = m_b^2/2$ as pointed out in [4,10,15] (the other electroweak penguin coefficients like c_7^{eff} and c_9^{eff} are not affected by this charm quark loop contribution in any significant amount). This enhancement of the strong penguin term increases the decay rates and bring the theoretical $B \rightarrow K\pi$ decay rates closer to the latest CLEO measurements. In the above expressions, the tree level amplitudes are suppressed relative to the penguin terms by the CKM factor $V_{ub}V_{us}^*/V_{tb}V_{ts}^*$ which can be approximated by $-(|V_{ub}|/|V_{cb}|) \times (|V_{cd}|/|V_{ud}|) \exp(-i\gamma)$ after neglecting terms of the order $O(\lambda^5)$ in the (bs) unitarity triangle. The $B \rightarrow K\pi$ decay rates then depend on the FSI rescattering phase difference $\delta = \delta_3 - \delta_1$ and the weak phase γ . In the following, we shall use the set of parameters of [7,20] which give $f_\pi = 133$ MeV, $f_K = 158$ MeV, $F_0^{B\pi}(0) = 0.33$, $F_0^{BK}(0) = 0.38$. We use $m_s = 120$ MeV, $|V_{cb}| = 0.0395$, $|V_{cd}| = 0.224$ and $|V_{ub}|/|V_{cb}| = 0.08$ [1]. At the moment, m_s is not known to a good accuracy, but a value around $100 - 120$ MeV inferred from $m_{K^*} - m_\rho$, $m_{D_s^+} - m_{D^+}$ and $m_{B_s^0} - m_{B^0}$ mass differences [21] seems not unreasonable. To show the factorisation predictions and the dependence of the branching ratios on the rescattering phase difference δ , we give, as an example, the CP-averaged $B \rightarrow K\pi$ decay rates in Fig.1 evaluated with a CKM value given by $\rho = 0.12$, $\eta = 0.34$ [7] corresponding to $\gamma = 70^\circ$.

As can be seen from Fig.1, all the $B \rightarrow K\pi$ decay modes for B^- and \bar{B}^0 , except the $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ mode, have branching ratios more or less in agreement with the latest CLEO data [2,3] which give, for the CP-averaged branching ratios

$$\begin{aligned}
\mathcal{B}(B^+ \rightarrow K^+\pi^0) &= (11.6_{-2.7-1.3}^{+3.0+1.4}) \times 10^{-6}, \\
\mathcal{B}(B^+ \rightarrow K^0\pi^+) &= (18.2_{-4.0}^{+4.6} \pm 1.6) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow K^+\pi^-) &= (17.2_{-2.4}^{+2.5} \pm 1.2) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow K^0\pi^0) &= (14.6_{-5.1-3.3}^{+5.9+2.4}) \times 10^{-6}.
\end{aligned} \tag{6}$$

The computed decay rates shown above could be larger with the form factors given in [22] and could bring the $B \rightarrow K\pi$ decay rates closer to the latest CLEO data.

We now turn to the test of factorisation in $B \rightarrow K\pi$ decays. The decay rates into a $K\pi$ final state is given by

$$\Gamma(B \rightarrow K\pi) = C|A_{K\pi}|^2 \tag{7}$$

where the subscript $K\pi$ refers to any of the decay modes for B^- and \bar{B}^0 and C is the usual phase space factor. By summing over the decay modes for B^- and for \bar{B}^0 respectively, we have in terms of A_1 , B_1 and B_3 ,

$$\begin{aligned}
\Gamma_{B^-} &= C \left[\frac{2}{3}|B_3|^2 + |A_1 + B_1|^2 \right] \\
\Gamma_{B^0} &= C \left[\frac{2}{3}|B_3|^2 + |A_1 - B_1|^2 \right].
\end{aligned} \tag{8}$$

where $\Gamma_{B^-} = \Gamma(B^- \rightarrow K^-\pi^0) + \Gamma(B^- \rightarrow \bar{K}^0\pi^-)$ and $\Gamma_{B^0} = \Gamma(\bar{B}^0 \rightarrow K^-\pi^+) + \Gamma(\bar{B}^0 \rightarrow \bar{K}^0\pi^0)$.

The quantities Γ_{B^-} and Γ_{B^0} are independent of the rescattering phase difference δ . They are given in the factorisation model as a function of the weak phase γ . Two other related quantities of interest obtained from the above Eq.(8) are

$$\begin{aligned}
r_b \mathcal{B}_{B^-} + \mathcal{B}_{B^0} &= 2C \left[\frac{2}{3}|B_3|^2 + |A_1|^2 + |B_1|^2 \right] \tau_{B^0} \\
r_b \mathcal{B}_{B^-} - \mathcal{B}_{B^0} &= 4C \operatorname{Re}(A_1^* B_1) \tau_{B^0}
\end{aligned} \tag{9}$$

which, together with one measured $B \rightarrow K\pi$ branching ratio, would enable us to determine the strength of the strong penguin contribution as well as its absorptive part and γ assuming factorisation for the small tree-level and electroweak penguin contributions, if the

rescattering phase difference δ could be inferred from the δ -dependent branching ratio and from other sources. In the above expression, τ_{B^0} is the B^0 lifetime and $r_b = \tau_{B^0}/\tau_{B^-}$,

Also, if all the four $B \rightarrow K\pi$ decay rates (CP-averaged) are measured with good accuracy, in particular with a precise measurement of the $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ branching ratio, the following quantities

$$R_1 = \frac{\Gamma_{B^-}}{\Gamma_{B^0}} \quad , \quad R_2 = \frac{\Gamma_{B^-}}{(\Gamma_{B^-} + \Gamma_{B^0})} \quad (10)$$

could also be used to test factorisation.

As the CP-averaged $B \rightarrow K\pi$ decay rates depend on γ , the computed partial rates Γ_{B^-} and Γ_{B^0} would now lie between the upper and lower limit corresponding to $\cos(\gamma) = 1$ and $\cos(\gamma) = -1$, respectively. As shown in Fig.2, where the corresponding CP-averaged branching ratios (\mathcal{B}_{B^0} and \mathcal{B}_{B^-}) for Γ_{B^-} and Γ_{B^0} are plotted against γ , the factorisation model values with the BWS form factors [20] seem somewhat smaller than the CLEO central values by about 10 – 20%. Also, $\mathcal{B}_{B^-} > \mathcal{B}_{\bar{B}^0}$ while the data gives $\mathcal{B}_{B^-} < \mathcal{B}_{\bar{B}^0}$ by a small amount which could be due to a large measured $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ decay rates. We note that smaller values for the form factors could easily accommodate the latest CLEO measured values, if a smaller value for m_s , e.g, in the range (80 – 100) MeV is used. What one learns from this analysis is that $B \rightarrow K\pi$ decays are penguin-dominated and the strength of the penguin interactions as obtained by perturbative QCD, produce sufficient $B \rightarrow K\pi$ decay rates and that factorisation seems to work with an accuracy better than a factor of 2, considering large uncertainties from the form factors and possible non-factorisation terms inherent in the factorisation model. With more precise measurements expected in the near future, it might be possible to have a detailed test of factorisation and a determination of δ and γ by comparing with experiments various relative branching ratios, to reduce uncertainties from form factors and CKM parameters. Other test of factorisation could also be done by looking for quantities which are independent of the strong penguin interactions. Infact, since the four $B \rightarrow K\pi$ decay rates depend on only three amplitudes A_1 , B_1 and B_3 , it is possible to derive a relation between the decay rates independent of A_1 . From the

following quantities,

$$\begin{aligned}
& \Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(\bar{B}^0 \rightarrow K^- \pi^+) = C_1 \\
& \times \left[\frac{1}{3} |B_3|^2 + (|A_1|^2 + |B_1|^2) - \frac{2}{\sqrt{3}} \text{Re}(B_3^* B_1 e^{i\delta}) \right] \\
& \Gamma(B^- \rightarrow K^- \pi^0) + \Gamma(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) = C_2 \\
& \times \left[\frac{4}{3} |B_3|^2 + (|A_1|^2 + |B_1|^2) + \frac{4}{\sqrt{3}} \text{Re}(B_3^* B_1 e^{i\delta}) \right]
\end{aligned} \tag{11}$$

where $C_1 = \frac{4}{3}C$ and $C_2 = \frac{2}{3}C$, we obtain

$$\begin{aligned}
\Delta &= \left\{ \Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(\bar{B}^0 \rightarrow K^- \pi^+) \right. \\
&\quad \left. - 2 \left[\Gamma(B^- \rightarrow K^- \pi^0) + \Gamma(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) \right] \right\} \tau_{B^0} \\
&= \left[-\frac{4}{3} |B_3|^2 - \frac{8}{\sqrt{3}} \text{Re}(B_3^* B_1 e^{i\delta}) \right] (C \tau_{B^0})
\end{aligned} \tag{12}$$

From Eq.(12), we see that Δ is independent of A_1 and hence is independent of the strong penguin term. It is given by the tree-level and electroweak contributions which are much smaller than the strong penguin term. As can be seen from Fig.2, where its values for $\delta = 0$ are plotted against γ . Δ is of the order $O(10^{-6})$ compared with \mathcal{B}_{B^-} and $\mathcal{B}_{\bar{B}^0}$ which are dominated by the strong penguin contribution and are in the range $(1.6 - 3.0) \times 10^{-5}$. As the variation with δ is negligible, Δ remains at the $O(10^{-6})$ level for other values of $\delta \neq 0$. Thus, to this level of accuracy, we can put $\Delta \simeq 0$. Eq.(12) becomes

$$r_b \mathcal{B}_{\bar{K}^0 \pi^-} + \mathcal{B}_{K^- \pi^+} = 2 [\mathcal{B}_{\bar{K}^0 \pi^0} + r_b \mathcal{B}_{K^- \pi^0}] . \tag{13}$$

This relation can be used as a test of factorization with more precise measurements of the CP-averaged branching ratios. Conversely, it can also be used to predict $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ in terms of the other measured branching ratios. From the latest CLEO data, with $\Delta \simeq 0$, Eq.(13) then gives $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0) = 0.60 \times 10^{-5}$. As can be seen, the large experimental errors prevent us from drawing any firm conclusion on the validity of factorisation, though, the above predicted central value for $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ is somewhat smaller than the CLEO data.

For another test of factorisation and a determination of γ we have derived a relation in the form of a ratio which is independent of the form factors and the CKM parameters. It is given by the ratio R of the two CP-averaged quantities as

$$R = \frac{[\mathcal{B}(B^- \rightarrow K^- \pi^0) + \mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-)]}{\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-) + \mathcal{B}(\bar{B}^0 \rightarrow K^- \pi^+)/r_b} . \quad (14)$$

Numerically, we find that terms proportional to $\cos(\delta)$ and $\sin(\delta)$ in R is of the order 10^{-7} and thus can be safely ignored. Thus R is a function of γ alone and can be used to determine γ as it does not suffer from the uncertainties in the form factors and in the CKM parameters. In Fig.3 we give a plot of R as a function of γ . As can be seen, it is not possible to deduce a value for γ with the CLEO data which gives $R = (0.80 \pm 0.25)$ as the theoretical prediction for R lies within the experimental errors. If we could reduce the experimental uncertainties to a level of less than 10%, we might be able to give a value for γ . Thus it is important to measure $B \rightarrow K\pi$ decays branching ratios to a high precision. It is interesting to note that the central value of 0.80 for R corresponds to $\gamma = 110^\circ$, close to the value $(113_{-23}^{+25})^\circ$ found by the CLEO Collaboration in an analysis of all known charmless two-body B decays with the factorisation model [2]. It seems that the CLEO data favors a large γ in the range $(90^\circ - 120^\circ)$. A large γ as shown in Fig.2, would increase the factorisation values for \mathcal{B}_{B^-} and \mathcal{B}_{B^0} which are given numerically by

$$\begin{aligned} \mathcal{B}_{B^-} &= (2.757 - 0.409 \cos(\gamma)) \times 10^{-5} , \\ \mathcal{B}_{B^0} &= (2.270 - 0.624 \cos(\gamma)) \times 10^{-5} . \end{aligned} \quad (15)$$

For the ratio R , we have

$$R = \frac{(2.651 - 0.393 \cos(\gamma))}{(3.253 - 0.652 \cos(\gamma))} \quad (16)$$

Also shown in Fig.3 are the ratio R_1 and R_2 defined in Eq.(10). As R_1 shows strong dependence on γ , a better way to determine γ would be to use R_1 rather than R when a precise value for $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ will be available.

Given $\gamma = 110^\circ$, all the $B \rightarrow K\pi$ branching ratios can be predicted in terms of the rescattering phase difference δ as shown in Fig.4. Comparing with Fig.1, we see that,

except for $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ which remains at the 6.5×10^{-6} level, the other branching ratios become larger with $\gamma = 110^\circ$ and closer to the CLEO data which indicate $B^- \rightarrow \bar{K}^0 \pi^-$ and $\bar{B}^0 \rightarrow K^- \pi^+$ are the two largest modes with near-equal branching ratios in qualitative agreement with factorisation. Fig.1 shows that these two largest branching ratios are quite apart, except for $\delta < 50^\circ$ while Fig.4 suggests δ should be large, in the range of $(40^\circ - 70^\circ)$. With a smaller $\gamma < 110^\circ$ and some adjustment of form factors, the current s quark mass and CKM parameters, it might be possible to accommodate these two largest branching ratios with a smaller δ . We note that the dependence of the four branching ratios shown in Fig.1 and Fig.4 are essentially the same and is given by $(4/3\sqrt{3}) \text{Re}(A_1 B_3^* \exp(i\delta))$, apart from the sign, as the interference term $\text{Re}(B_1 B_3^* \exp(i\delta))$ is much smaller than $\text{Re}(A_1 B_3^* \exp(i\delta))$.

We note that we have also considered a possible contribution from inelastic rescattering effects as an additional small absorptive contribution A_i to A_1 from $D D_s^*$ and other intermediate states to the S -matrix unitarity relation. We find that the variation of Γ_{B^-} and Γ_{B^0} as a function of A_i is negligible. For this reason, we have set $A_i = 0$. Also, since the theoretical values for the decay rates shown above show qualitative agreement with the measured values, the strong penguin terms with enhancement by the internal c -quark loop seem to produce sufficient decay rates, a large dispersive inelastic contribution would not be needed in $B \rightarrow K\pi$ decays.

The CP-asymmetries, plotted against δ as shown in Fig.5, are given by

$$A_{\text{CP}} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad (17)$$

where Γ is the decay rate. The predicted CP-asymmetry A_{CP} for the $B \rightarrow K\pi$ decay modes are in the range $\pm(0.04)$ to $\pm(0.3)$ for the preferred values of δ in the range $(40 - 70)^\circ$ mentioned above. The latest CLEO measurements [23] however, do not show any large CP-asymmetry in $B \rightarrow K\pi$ decays, but the errors are still too large to draw any conclusion at the moment.

In conclusion, factorisation with enhancement of the strong penguin contribution seems to describe qualitatively the $B \rightarrow K\pi$ decays, although the predicted $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ is

below the measured value. Further measurements will enable us to have a more precise test of factorisation and a determination of the weak angle γ from the FSI phase-independent relations we shown above.

Acknowledgments

We would like to thank Dr Kwei-Chou Yang for pointing out an error in our initial numerical calculations.

REFERENCES

- [1] For a review of the CKM model, see, e.g,
Particle Data Group, C. Caso *et al.*, Eur. Phys. Jour. **C3**, 103 (1998).
- [2] CLEO Collaboration, Y. Kwon *et al.*, CLEO-CONF 99-14.
- [3] CLEO Collaboration, D. Cronin-Hennessy *et al.*, CLEO 99-18, hep-ex/0001010.
- [4] R. Fleischer and T. Mannel, Phys. Rev. **D57**, 2752 (1998).
- [5] M. Neubert, Talk given at the Heavy Flavor 8, Southampton, UK, 1999, hep-ph/9910530.
- [6] N. G. Deshpande, X. G. He, W. S. Hou, and S. Pakvasa, Phys. Rev. Lett. **82**, 2240 (1999).
- [7] A. Ali, G. Kramer, and C. Lü, Phys. Rev. **D58**, 094009 (1998).
- [8] A. Buras, M. Jamin. M. Lautenbacher, and P. Weisz, Nucl. Phys. **B400**, 37 (1993); A. Buras, M. Jamin, and M. Lautenbacher, *ibid.* **B400**, 75 (1993).
- [9] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Phys. Lett. **B316**, 127 (1993) ; M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Nucl. Phys. **B415**, 403 (1994).
- [10] R. Fleischer, Z. Phys. C **58**, 483 (1993) ;**62**, 81 (1994).
- [11] G. Kramer, W. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994).
- [12] N. G. Deshpande and X. G. He, Phys. Lett. **B336**, 471 (1994).
- [13] A. Buras and L. Silvestrini, Nucl. Phys. **B520**, 293 (1999).
- [14] M. Neubert and B. Stech, hep-ph/9705292, to appear in *Heavy Flavours II*, eds. A. J. Buras and H. Lindner, World Scientific, Singapore.

- [15] J.-M. Gérard and W.S. Hou, Phys. Rev. Lett. **62**, 855 (1989); Phys. Rev. **D43**, 2909 (1991).
- [16] A. N. Kamal and T. N. Pham, Phys. Rev. **D50**, 6849 (1994).
- [17] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999);
M. Beneke, Talk given at the Heavy Flavor 8, Southampton, UK, 1999, hep-ph/9910505.
- [18] A. N. Kamal and T. N. Pham, Phys. Rev. **D50**, 395 (1994).
- [19] M. Neubert, V. Rieckert, B. Stech and Q. P. Xu, in *Heavy Flavours*, eds. A. J. Buras and H. Lindner, World Scientific, Singapore (1992).
- [20] M. Bauer, B. Stech and M. Wirbel, *Z. Phys. C* **34**, 103 (1987).
- [21] P. Colangelo, M. Ladisa, G. Nardulli and T. N. Pham, Phys. Lett B **416**, 208 (1998).
- [22] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281** 145 (1997).
- [23] CLEO Collaboration, S. Chen *et al.*, CLEO 99-17, hep-ex/0001009.

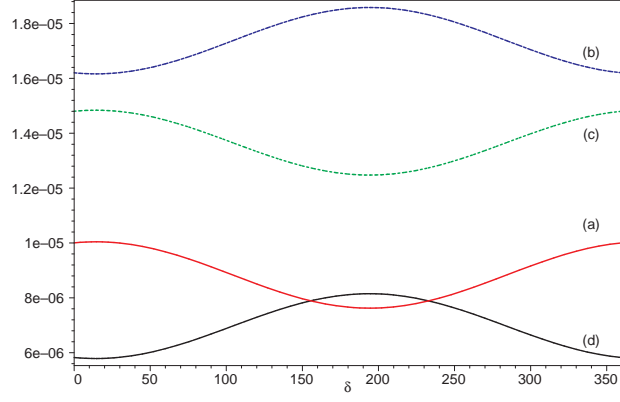


FIG. 1. $\mathcal{B}(B \rightarrow K\pi)$ vs. δ for $\gamma = 70^\circ$. The curves (a), (b), (c), (d) are for the CP-averaged branching ratios $B^- \rightarrow K^- \pi^0$, $\bar{K}^0 \pi^-$ and $\bar{B}^0 \rightarrow K^- \pi^+$, $\bar{K}^0 \pi^0$, respectively.

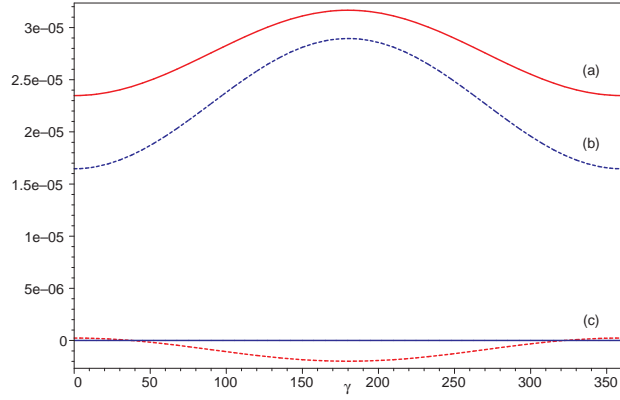


FIG. 2. \mathcal{B}_{B^-} (a), $\mathcal{B}_{\bar{B}^0}$ (b), Δ (c) vs. γ

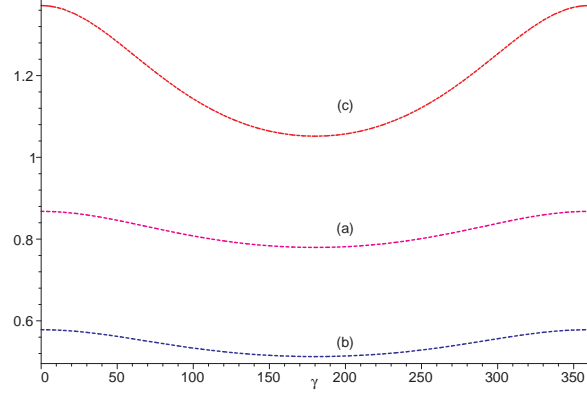


FIG. 3. The curves (a), (b), (c) are for R , $R2$, $R1$ respectively

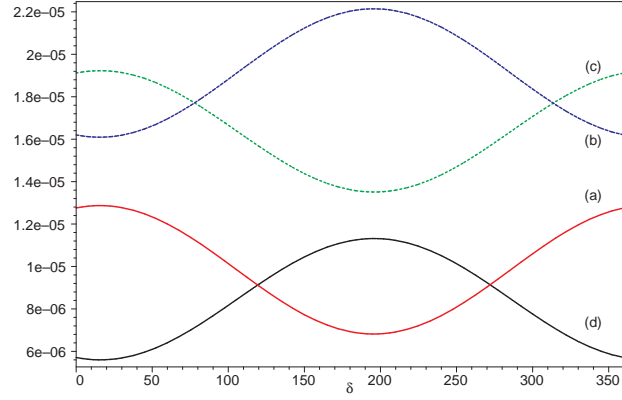


FIG. 4. $\mathcal{B}(B \rightarrow K\pi)$ vs. δ for $\gamma = 110^\circ$. The curves (a), (b), (c), (d) are for the CP-averaged branching ratios $B^- \rightarrow K^- \pi^0$, $\bar{K}^0 \pi^-$ and $\bar{B}^0 \rightarrow K^- \pi^+$, $\bar{K}^0 \pi^0$, respectively.

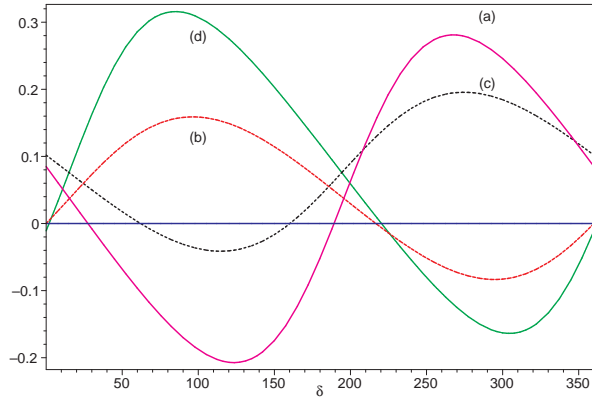


FIG. 5. The asymmetries vs. δ for $\gamma = 110^\circ$. The curves (a), (b), (c), (d) are for $As_{B^- \rightarrow K^- \pi^0}$, $As_{B^- \rightarrow \bar{K}^0 \pi^-}$, $As_{\bar{B}^0 \rightarrow K^- \pi^+}$, $As_{\bar{B}^0 \rightarrow \bar{K}^0 \pi^0}$,